Spring 2025 CISC-1100-C01 Midterm

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Abstract

Covering chapters 1–4 of Fundamentals of Discrete Structures by Lyons et al.

1. In a survey,
   1 person likes ketchup, pickles, and mustard on their hamburgers.

   3 people like ketchup and pickles on their hamburgers.
   4 people like pickles and mustard on their hamburgers.
   5 people like ketchup and mustard on their hamburgers.

   12 people like ketchup on their hamburgers.
   12 people like pickles on their hamburgers.
   15 people like mustard on their hamburgers.

   Draw a Venn diagram, with sets named K, P, and M. How many people were in the whole survey? (This example is taken from the notes of Arthur G. Werschulz.)

2. Write the following summation using sigma notation. (You don't have to figure out that the total is 28.)
   $$1+2+3+4+5+6+7$$

3. Let $A = \{0, 1, 2\}$.
   Within a pair of {curly braces}, list all the elements of the power set of A.
   How many elements are there?

4. Let A and B be sets. Is it always true, or always false (or sometimes true, or sometimes false) that
   $$(A \cup B)' = A' \cap B'$$

5. Let $A = \{a, b\}$.
   Let $B = \{b, c\}$.
   (a) Within a pair of {curly braces}, list the elements of the set $A \cup B$.
   (b) Within a pair of {curly braces}, list the elements of the set $A \cap B$.
   (c) Within a pair of {curly braces}, list the elements of the set $A - B$.
   (d) Within a pair of {curly braces}, list the elements of the set $A + B$. In case you don't remember, this set is defined to be $(A - B) \cup (B - A)$.
   (e) Within a pair of {curly braces}, list the elements of the set $A \times B$.

6. Insert a pair of parentheses into each of the following expressions to indicate the order in which the operators are evaluated. For example, let if a, b, c be numbers. Then
   $$a + b \times c$$
   means the same thing as
   $$a + (b \times c)$$
   Now let p, q, r be propositions.

(a) $p \vee q \wedge r$

(b) $p \vee q \oplus r$

(c) $p \wedge q'$

(d) $p \wedge q \wedge r$

(e) $p \iff q \iff r$

(f) $p \implies q \implies r$

7. Fill in the truth table for the following logical operators:

(a)

p	q	$p \oplus q$
F	F	
F	T	
T	F	
T	T	

(b)

p	q	$p \implies q$
F	F	
F	T	
T	F	
T	T	

(c)

p	q	$p \iff q$
F	F	
F	T	
T	F	
T	T	

8. Let p and q be propositions. Prove that

$$(p \oplus q) \equiv (p \wedge q)' \wedge (p \vee q)$$

and then paraphrase in jaunty, informal English what the expression $(p \wedge q)' \wedge (p \vee q)$ means.

9. Write each of the following natural numbers as a three-bit binary number:

(a) zero

(b) one

(c) two

(d) three

(e) four

(f) five

(g) six

(h) seven

Extra credit. Who was the first person generally known to have written numbers in binary?

10. Now that you have written the first eight natural numbers in binary, pick one of them and store its three bits into the three variables p_0, p_1, p_2.
Put the one's place of your number into p_0.
Put the two's place of your number into p_1.
Put the four's place of your number into p_2.

    Then
    Let $q_0 = (p_0)'$.
    Let $b = q_0$.
    Let $q_1 = p_1 \oplus b$.
    Let $b = (b \implies q_1)'$.
    Let $q_2 = p_2 \oplus b$.

    What three-bit binary number is now held in the variables q_0, q_1, q_2?
    q_0 is the one's place.
    q_1 is the two's place.
    q_2 is the four's place.
    What does this machine do to the number you started with?
    Extra credit: why did I name the variable b?

11. What is the difference between $\mathbb{N}$ and $\mathbb{Z}$?

12. Assume you have a web page containing the following text:

    `New York Times`

    Make this text a link leading to the destination

    `https://www.nytimes.com/`

    Make sure your punctuation and capitalization is complete and correct.

13. Describe this sequence

$$1, 2, 6, 24, 120, 720, 5040, 40320, 362880, 3628800, \ldots$$

    using

    (a) the recursive method
    (b) the closed method

14. What is the value of

$$\sum_{i=1}^{50} i$$

15. What is the value of

$$\sum_{i=1}^{4} (2i - 1)$$

    Hint: draw a picture made of lots of little squares. How many little squares are there?

16. Let p be a proposition.
What is the dual of each of the following?

    (a) $p \vee p' \equiv T$
    (b) $p \wedge p' \equiv F$

17. Let $A = \{a, b, c, d\}$ be a set. Consider the following relation on A. (The color is just to make it easier to read.)

	a	b	c	d
a	F	T	T	T
b	T	F	T	T
c	T	T	F	T
d	T	T	T	F

(a) Is this relation reflexive? How do you know?

(b) Is this relation symmetric? How do you know?

(c) What relation is this?

18. Let $A = \{0, 1, 2\}$ be a set of natural numbers. Consider the following relation on A. (The color is just to make it easier to read.)

	0	1	2
0	F	T	T
1	F	F	T
2	F	F	F

(a) Is this relation transitive? How do you know?

(b) What relation is this?

19. Let A = $\{0, 1, 2\}$ be a set. Consider the relation on A pictured in the following table. The table is one way to represent this relation. ("Represent" means to capture on paper every fact there is to know about this relation.)

	0	1	2
0	T	T	T
1	F	T	T
2	F	F	T

Another way to represent this relation is to list all the pairs of members of A for which this relation is true. Here are the six such pairs. (I used color only for consistency with the above table.)

$$0 \leq 0$$
$$0 \leq 1$$
$$0 \leq 2$$
$$1 \leq 1$$
$$1 \leq 2$$
$$2 \leq 2$$

What would be another way to represent this relation? Two hints: (1) Please don't draw a diagram with nodes and arrows. (2) Your instructor is fascinated by the idea of mathematical reductionism.

20. Let p be a proposition. Is $p \wedge p'$ ever true?